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Corrigendum

Correction to "An existence theorem for some semilinear elliptic systems" [J. Differential Equations 226 (2006) 572–593]

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Two corrections follow. The author gratefully thanks Dr. Jaeyoung Byeon of the University of Wisconsin for raising both points. The two issues are perhaps particularly important, as they are central to the development of a practical computation algorithm based on the method.

1. A constant

Throughout, brackets denote equation numbers in the above-referenced paper.

The last two terms in the right side of [3.12] need to be multiplied by a constant, depending on the domain Ω and the precise form of the boundary conditions, as reflected in the constant c in the right side of [3.10].

Thus the factor $1/2$ appearing in the right side of [3.13] needs to be replaced by a constant c, depending on α, Ω, and the boundary conditions, and the corresponding changes made thereafter. This has no significant effect of the subsequent analysis.

2. Infinitely many "restarts"

The algorithm as described in Section 5 could in principle fail because of the occurrence of an unlimited number of "restarts" as described in [5.20–5.22]. Here we give sufficient conditions that if such should occur, we obtain a solution as the limit of a subsequence of restart points.

Let $\{u_j\}_{j=1}^{\infty}$ be an infinite sequence of functions in X at which a restart occurs, corresponding to the points $\underline{u}$ in [5.15].

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Let ϕ_{j-} be the corresponding function ϕ in Q as u_j is approached, i.e. the corresponding ω in [5.16], and ϕ_{j+} the corresponding function $\phi(0)$ for beginning the next cycle, as appearing in [5.21, 5.22].

Denote by

$$K_j = K_{\phi_{j-}}(u_j) = K_{\phi_{j+}}(u_j) \tag{2.1}$$

the corresponding values $\underline{K}$ in [5.18] and [5.22], and by

$$\tilde{H}(u,\phi) = \int_{\Omega} K'_{\phi}(u) P_1 K'_{\phi}(u) \tag{2.2}$$

for any u in X, ϕ in Q.

Theorem. *Assume $n \leqslant 5$; choose $N < 1/2$ in* [2.2, 2.3], *and assume that each ϕ_{j+} is chosen so that*

$$\operatorname*{var}_{\Gamma} \phi_{j+} \leqslant c \tag{2.3}$$

and

$$\tilde{H}(u_j,\phi_{j+}) \geqslant \frac{1}{c} \sup_{\phi:\, K_{\phi}(u_j)=K_{\phi_{j-}}(u_j)} \tilde{H}(u_j,\phi) \tag{2.4}$$

for some c independent of j.

Then if an unlimited number of restarts occur, extracting a subsequence if necessary,

$$u_j \to U \tag{2.5}$$

strongly in X as $j \to \infty$, and the limit U is a solution of $\rho(U) = 0$.

Remark. Choosing c sufficiently large, (2.3), (2.4) are readily satisfied. Indeed, the functions θ determined from [5.3] are of uniformly bounded variation for u in a bounded region of X.

Proof. We outline the proof, omitting tedious, elementary details.

Differentiating [5.6] with respect to s, we get an estimate for $|H_s|$.

We use $N < 1/2$ to get $\psi_{\epsilon,uu}$ bounded, continuing the derivatives as in [2.16–2.18], and use $n \leqslant 5$ so that X is precompact in $L_3(\Omega)$. Using [4.14] and [5.4], we get

$$\left|H_s(s)\right| \leqslant cH(s) \tag{2.6}$$

with c a generic constant here and below. From (2.6) we see that H decreases at most exponentially as a function of s within each cycle, and thus from [5.11], (2.1), (2.2)

$$K_j - K_{j+1} \geqslant c\tilde{H}(u_j,\phi_{j+}) \tag{2.7}$$

with $c > 0$. As K_j are decreasing and bounded below, from (2.7) necessarily

$$\tilde{H}(u_j, \phi_{j+}) \to 0 \tag{2.8}$$

as $j \to \infty$.

The u_j are bounded in X independently of j, so extracting a subsequence as necessary, as $j \to \infty$ we have

$$u_j \rightharpoonup U \tag{2.9}$$

weakly in X;

$$u_j \to U \tag{2.10}$$

strongly in $L_2(\Omega)$;

$$\phi_{j+} \to \underline{\phi} \tag{2.11}$$

strongly in $L_\infty(\Gamma)$.

From [3.17], using (2.9)–(2.11), there is a distribution g in $X^* \times \Gamma$ such that as $j \to \infty$,

$$T_\gamma u_j - (1 + \gamma)\Delta u_j \to g. \tag{2.12}$$

Then from [4.14] and (2.2), using (2.9)–(2.12) it follows that as $j \to \infty$,

$$\tilde{H}(u_j, \phi_{j+}) - \left(1 + \int_\Gamma \gamma\underline{\phi}\right)^2 \int_\Omega |\nabla u_j|^2 \to \tilde{H}(U, \underline{\phi}) - \left(1 + \int_\Gamma \gamma\underline{\phi}\right)^2 \int_\Omega |\nabla U|^2. \tag{2.13}$$

As $\tilde{H}(U, \underline{\phi})$ is nonnegative from (2.2), it follows that (2.8) and (2.13) are compatible only if

$$\limsup_{j\to\infty} \int_\Omega |\nabla u_j|^2 \leqslant \int_\Omega |\nabla U|^2. \tag{2.14}$$

Now (2.5) follows from (2.9) and (2.14). From (2.8), using (2.5), (2.11), and the assumption (2.4),

$$\sup_{\phi:\, K_\phi(U) = K_{\underline{\phi}}(U)} \tilde{H}(U, \phi) = 0. \tag{2.15}$$

From (2.3) and (2.11), the function $\underline{\phi}$ cannot be either $\delta(\gamma - 1)$ or $\delta(\gamma - 2)$. Thus using (2.15) and appealing to Lemma 4.2, necessarily U is a solution. □